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AAE 550, Final Project Report

Geometry Optimization of a Scramjet Combustor

**Objective**

This project aims to optimize the geometry of a rectangular profile scramjet combustor for maximum thrust by varying the contour of one of the combustor walls. For simplification purposes, it is assumed that the nozzle perfectly and isentropically expands the resultant flow, producing the maximum thrust possible for a given combustor design.

For this project, it is assumed the scramjet is operating at a 20 km altitude, cruising at Mach 5, injesting 100 kg/s of air with a 0.3 pressure recovery factor and a 1/3 Mach recovery factor through the inlet and isolator system. The combustor has a rectangular cross section with a 5:1 aspect ratio and is 3 meters long. A single wall is split into 20 segments whose angles can be individually altered to create a contour. Hydrogen fuel is used at an equivalency ratio of 0.9. Further details on the fuel injection regime and properties can be found in the appendix.

The maximum jet thrust produced by the combustor is a function of the exit conditions of the combustor, which themselves are a function of the combustor properties. For easier representation, the thrust equation is reduced to the following form:

In this representation, is the angle of the *nth* wall section that together compose the design variable matrix , and is the collection of parameters that defines the initial starting conditions. Transforming the problem from a maximization to minimization problem, the objective becomes:

**Constraints**

Constraints were placed on the model to keep the solution feasible and practical. First, and consecutive wall segments along the contour must be angles within 5 degrees of each other to prevent large changes in combustor geometry that could cause flow separation or strong shocks in the flow. Additionally, the wall angles were bounded to greater than or equal to 0, but less than or equal to 25 degrees. Second, at any point in the flow, the Mach number must be greater than or equal to 1.05. Perturbations in the upstream flow could cause the Mach number to drop, and if Mach 1 is reached, the flow chokes and risks unstarting the inlet system, causing a dramatic loss of thrust and, most likely, a failed mission. The exit Mach from the combustor must be at least 1.15 so that afterburning remains feasible without running a great risk of thermally choking the flow. Finally, the maximum temperature in the combustor is limited to 3000 K in order to keep heat loads manageable.

The nonlinear constraint functions derived from the physical constraints above are as follows:

As before, is the angle of a wall section and is the design variable of interest. is the Mach number at the *mth* integration step, and likewise, is the static temperature at the *mth* integration step. The total number of nonlinear constraint equations depends upon the total number of wall sections (whose angle can be varied) and the total number of integration steps along the combustor. In a model with 20 wall sections and an integration step of 1 cm produces 679 constraint equations. The lower and upper bounds for wall angles are linear constraints, and as such, are simple and not represented in the above equations.

**Scramjet Combustor Model**

The scramjet combustor model is given by Shapiro [1] as a 1-D analysis, of which the main relations are given below.

More detail regarding these equations are provided in the appendix. The changing values of the specific heat (*cp*) and the ratio of specific heats () are calculated using NASA’s Chemical Equilibrium with Applications (CEA) program, which is a chemical property solver developed for determining the thermodynamic properties and physical state of complex mixtures of reactants. The effects of cooling at the wall and wall friction were ignored for simplification purposes. To determine combustor properties, the above equations are numerically integrated while using CEA to update thermodynamic properties at each integration step.

**Methodology**

The Sequential Quadratic Programing (SQP) optimization algorithm was used for a multitude of reasons. With the combustor taking significant computation time to solve, the fast and relatively efficient nature of SQP will make the problem more feasible in the allotted time. There is a significant chance the optimization procedure will venture into unfeasible territory easily, which SQP can handle. Additionally, the non-linear nature of the flow field constraints will be better represented by SQP than other strictly linear methods. There was a risk of stepping outside of the continuous domain of the objective function, which occurs when the Mach number in the combustor drops below 1, but the constraints put in place due to the physical ramifications of this scenario prevent such discontinuities from occurring. Matlab’s “fmincon” function with the SQP algorithm was used in practice.

The scramjet combustor model was also developed in Matlab. Calculating the gradients of the objective and constraint functions was practically impossible to do analytically, and extremely time consuming when done numerically. To make gradient calculations more feasible, a significant amount of effort was put into making the model more computationally efficient as well as parallelizing numeric gradient calculations. All gradients were calculated using the forward difference method in order to minimize the number of objective function calls. The model was also developed to have flexible integration fidelity so that, should optimization become intractable within the allotted project time, a low-fidelity solution could be found.

**Results**

**Final Remarks**

Works Cited

Ascher H. Shapiro, “The Dynamics and Thermodynamics of Compressible Fluid Flow”, Vol. 1

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